Correlation-based approach to online map validation

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***Abstract*— High definition (HD) maps are one of the key technologies supporting autonomous-driving vehicles (ADVs). Especially in urban scenarios, the field of view of sensors is often limited, and HD map provides critical information about upcoming road environmental data. Maps used for ADVs are high resolution with centimeter-level accuracy and their correctness is fundamental when analyzing the safety of upcoming maneuvers.**

This paper proposes an approach for online map validation (OMV) based on spatial and temporal correlation of smart-sensors. Smart sensors are capable of analyzing the validity of regions of the map independently from one another. Results from the sensors are then fused over multiple regions and time samples for providing a unified view to software components deciding on upcoming maneuvers which areas of the maps are consistent with sensor data and which are not.

I. Introduction

ADVs promise to radically change and improve the way mobility affects society [1], [2]. Safety is one of the first features that ADVs aim for. With the goal of reducing the possibility of missing safety-relevant elements around the ADV, measurements from sensors that rely on heterogeneous physical phenomena are often fused [3], [4], [5], [6]. Maps can be considered as sensors whose field of views (FOVs) are not affected by traffic conditions and whose measurements have unknown and potentially extensive latency, e.g., days, or weeks. The effectively unlimited FOV of maps and their extremely high accuracy, make them an exceptional sensor for speculatively planning upcoming maneuvers and for supporting the creation of an environment model in case of dense traffic. The unknown latency of the map data, however, largely limits their usability and make it imperative to use OMV approaches. Map validation needs to consider both geometrical and semantic data, e.g., driving direction and maximum speed limit.

This paper focuses on the validation of geometrical data by relying on spatial and temporal correlation of measurements. Section II introduces the problem of map validation for ADV. Section III discusses relevant research activities in the area of OMV and of online road model (ORM). Section IV discusses the approach we propose and section V provides additional details by means of an example. Section VI discusses how low-level data from sensors can be integrated in the proposed framework. Finally, section VII reviews the proposed approach and its limitations and discusses future research directions.

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II. The Problem of Map Validation

We divide the problem of map validation into four cases: (i) *global map invalidation*, (ii) *local map invalidation*, (iii) *map validation*, and (iv) *map correction*. In the first case, the goal of a map validation component is to identify that a map contains invalid data. There is no attempt to identify which parts of the map are invalid. The second case focuses on the identification of the geographical area in which invalid data are detected. The third case considers a map invalid until its content has been validated with sensor measurements. In the last case, those areas of the map that have been identified as invalid must also be corrected by data coming at run-time from sensors. Depending on the specific level of automatic driving and on requirements on data integrity and accuracy, either of the cases described above might be appropriate. For example, if the car can return the control to the driver at any time, e.g., such as in the case of automation level 3, a global map invalidation approach might be sufficient.

In this paper we focus on the case (ii), i.e. the invalidation of a map and the identification of which parts of the map are invalid. Furthermore, we focus on the case where an ADV must be able to partially cope with an invalid map, e.g., by not entering those areas that contain invalid map data and plan alternative routes instead.

III. Previous Work

Although the problem of OMV is not completely solved in the field of ADVs, very few research efforts have been made for addressing this topic. Most academic work has been focusing on feature extraction from sensor data and on on-line map creation via Simulateneous Localization and Mapping (SLAM) approaches [7], [8]. In this class of approaches a map is created online from sensor data and then compared against HD map data. Landsiedel and Wollherr [9] propose the integration of a 3D metric environment representation with the semantic knowledge from open data. Tanzmeister *et al.* [10], [11] focus on estimating the drivable area using vehicle motion in unknown environments. With such a path planning-based approach it is possible to obtain realistic road-course estimations. Hasberg and Hensel [12] describe an approach for fusing noisy sensor data in a parametric representation of the lane by means of a probabilistic approach. This framework could be used for validation purposes by including a reference map in the scheme.

Machine learning (ML) approaches can also be used in order to classify and estimate the road that can then be used to validate the map. In [13], Fernandez *et al.* generate a model of a complex urban scenario using the information provided by a digital navigation map and a vision-based sensing module. In this approach, a road pixel segmentation is performed by relying on a convolutional neural network (CNN) fused with map data. Mayer *et al.* [14] use a deep neural network (DNN) approach applied to an image for extracting lane boundaries of the ego lane including the semantic information about adjacent lanes.

These approaches, although helpful in addressing the problem of OMV do not meet the requirements for high-accuracy needed in the validation of HD maps. In particular, sensor uncertainty at tens of meter in front of the vehicle is already order of magnitude higher than the accuracy of HD maps and hence, provide little value in validating areas of HD maps before they are used for deciding upon upcoming maneuvers. Furthermore, approaches based on gaussian process (GP), tend reduce the accuracy of the underlying model for the benefit of increasing computational efficiency.

This paper discusses an approach for computing the validity of a map by relying on sensor measurements as well as on expected correlation among measurements. In literature, many examples can be found in which the concept of spatial correlation has been applied to a map. Blanco *et al.* [15] introduce a probabilistic method to learn a gas distribution model of planar environments, given a sequence of localized gas sensor readings. In their work, a simple sensor model, leads to the derivation of an efficient implementation of a Kalman Filter. An alternative approach is presented by Ke Sun *et al.* [16]. The focus of the latter is to provide a model that captures the occupancy correlation of map elements, providing an efficient approximation of GP [17] classification using a Kalman-filter-like structure.

We consider the case where random variables are discrete, e.g., a map is valid, invalid, or unknown and hence, no approaches similar to GP are made. Although the overall algorithm might require higher computational resources, we believe that it could still be useful for industrial applications for which the compromise between accuracy and computational resources favourites the first point. Furthermore, we believe that the proposed approach better captures the complex level of information provided by today's sensors, which are often equipped with complex classifiers able to independently from one another verify if a map is valid or not. Finally, the proposed approach can also be extended in order to include input from sensors which provide low-level data only, e.g., position of traffic signs and or lanes. An example in this direction is discussed in section VI.

IV. A Correlation-based Approach

We suppose that the ADV is equipped with several sensors, each capable of observing a part of the environment around the vehicle. Furthermore, we assume the existence of a labelling system, which discretizes the environment around the vehicle into non-overlapping cells. This discretization of the environment does not necessarily need to be uniform. For example, a road can be discretized laterally along lanes and longitudinally in chunks of appropriate size, e.g., 10 m. Within the labelling system, e.g., ML-based approaches may be used for labelling these regions, using the available sensor data. Labels determine the expected level of validity, or of invalidity, of the associated region.

Due to the limited FOVs and occlusion of the sensors, we expect that certain regions cannot be labeled directly by smart sensors. In order to define a probability of validity over every region of a map a correlation function is needed, such as the one introduced in eq. (4). The correlation defines the probability that a region takes a certain label based on the labels given to the other regions. This assumption represents the case where if a region is invalid, then nearby regions are also likely to be invalid. The concept of distances between regions can also be extended in order to account for temporal distances between measurements. In this case, a region that was recently observed by a sensor and classified as valid, is likely to remain valid also in the near future. With p_i we denote the i^{th} region of the map and with $||p_i - p_j||$ we denote the distance between the region p_i and p_j. Distance can consider both spatial and temporal differences among regions.

With $m(p_i)$ we denote the state of the map (Valid or Invalid) on the i^{th} region. The actual validity of the map is unknown, but the prior probability of the map validity is known. Note that, since the regions are divided over time and space, the a priori distribution ranges over both time and space. With $s(p_i)$ we denote the label assigned by the labelling system to the i^{th} region. The problem of map validation addressed in this paper is the calculation of the posterior distribution of the i^{th} region

$$P\left[m(p_i)|s(p_{j1}),\ldots,s(p_{jn})\right], \tag{1}$$

where $\{s(p_{j1}),\ldots,s(p_{jn})\}$ is a set of labels associated to the regions $p_{j1},\ldots,p_{jn}$. The latter are a subset of n regions where information about the measured validity of the map is available. The correlation defines the shape of

$$P\left[m(p_i)|m(p_{j1}),\ldots,m(p_{jn})\right], \tag{2}$$

for each region p_i and every subset of regions $\{p_{j1},\ldots,p_{jn}\}$.

V. Example

In this section we discuss the proposed approach by means of an example in which additional assumptions and simplifications are made which lead to a clearer explanation of the method. We assume the labelling system uses three labels only: *Valid*, *Invalid*, and *Unknown*. For each observed region, the associated validity label uniquely depends on the features observed in that region. For example, if a camera is used for validating a region based on the measured positions of lane marking, the validity of a specific region only depends on the lane position in that specific region and not in nearby regions. This remark can be formalized as the following

Assumption: Given a subset $\mathcal{M}$ of map regions and a subset $\mathcal{S}$ of labels over all regions that do not include the label given to the i^{th} region, the conditional probability of

TABLE I: Characteristic of the sensors.

$s(p_i)$	$m(p_i)$	$P[s(p_i)\|m(p_i)]$
Valid	Valid	0.85
Invalid	Valid	0.1
Unknown	Valid	0.05
Valid	Invalid	0.1
Invalid	Invalid	0.85
Unknown	Invalid	0.05

associating a label to the i^{th} region only depends on the validity of the region itself

$$P[s(p_i)|\mathcal{M}, m(p_i), \mathcal{S}] = P[s(p_i)|m(p_i)] \tag{3}$$

for all regions p_i.

The equation above defines the *characteristics* of the labelling systems, i.e. the probability of associating a specific label once the actual validity of the map in the i^{th} region is known. Table I shows the specific characteristics of the sensors used in this example.

We further assume that the correlation among map regions only depends on their distance, in particular it tends to 0 as the distance tends to infinity and tends to the function γ as the distance tends to 0. The function γ is defined in table II and accounts for consistency between the validity of the map labels.

There is no unique way to define eq. (2), since there are infinite ways to take into consideration the different sources of information and their distance (temporal and spatial). A suitable function representing the correlation among n regions is

$$\begin{aligned} &P\left[m(p_i)|m(p_{j1}),...m(p_{jn})\right] = \\ &= S\Big(\sum_{h=j1}^{jn}\Big[L\left(P\left[m(p_i)|m(p_h)\right]\right) - L\left(P\left[m(p_i)\right]\right)\Big] + \\ &\qquad\qquad L\left(P\left[m(p_i)\right]\right)\Big), \end{aligned} \tag{4}$$

where $S(x)$ is the Sigmoid function and $L(x)$ is its inverse

$$S : \mathbb{R} \to (0,1) \qquad L : (0,1) \to \mathbb{R}$$
$$x \mapsto \frac{e^x}{e^x+1} \qquad x \mapsto \ln\left(\frac{x}{1-x}\right)$$

and $\ln(\cdot)$ is the logarithm with base e.

For every pair of regions, the conditional probability $P\left[m(p_i)|m(p_j)\right]$ is defined as

$$\begin{aligned} P\left[m(p_i)|m(p_j)\right] = &\Big(1 - k(||p_i - p_j||)\Big) \cdot P\left[m(p_i)\right] + \\ &k(||p_i - p_j||) \cdot \gamma(m(p_i), m(p_j)) \end{aligned} \tag{5}$$

where $k(\cdot)$ is a monotonic function decreasing with the distance of p_i and p_j and it is limited to the interval $[0,1]$. There are multiple ways to define $k(\cdot)$ and finding a "good" function that is able to guarantee high performances with low computational cost can be challenging.

In order to satisfy these requirements it is important to define a correlation function that has non-zero value only for

TABLE II: Conditional probability of the map.

$m(p_i)$	$m(p_j)$	$\gamma\left(m(p_i), m(p_j)\right)$
Valid	Valid	1
Invalid	Valid	0
Valid	Invalid	0
Invalid	Invalid	1

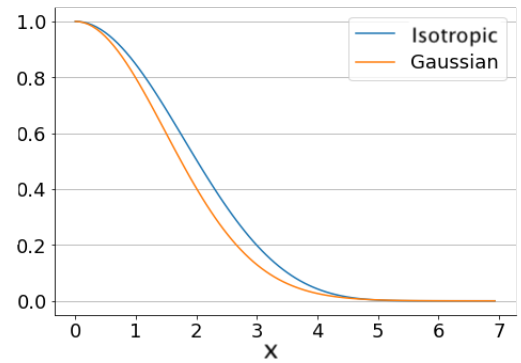


Fig. 1: Example of isotropic covariance function and Gaussian Kernel function as function of the variable x.

a small portion of the space. One example is the following isotropic covariance function proposed by Storkey in [18]:

$$k(\cdot) = \begin{cases} \dfrac{(2\pi - \Delta(\cdot))\left(1 + (\cos\Delta(\cdot)\right)/2}{3\pi} + \\ \qquad\qquad + \dfrac{\frac{3}{2}\sin\Delta(\cdot)}{3\pi}, & \text{if } \Delta < 2\pi \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

where $\Delta = \beta_{corr}||p_i - p_j||_2$, for $\beta_{corr} > 0$. This function closely resembles a Gaussian kernel $k(||p_i - p_j||) = e^{\frac{-\beta_{corr}||p_i - p_j||^2}{2\pi}}$, but has zero value for distances larger than $2\pi/\beta_{corr}$. In fig. 1 the isotropic covariance function is compared with the standard Gaussian Kernel.

By varying the value of β_{corr}, it is possible to vary the range of influence of temporally or spatially adjacent labels. In particular, larger values of β_{corr} tend to reduce the influence on adjacent labels, whereas smaller values of β_{corr} tend to increase the influence on adjacent labels.

The proposed correlation function satisfies our assumptions: it tends to 0 as the distances among regions tends to infinity and converges to the function γ as the distances tend to 0. A proof that this function converges to the function γ as the distance among regions tends to 0 is given by Fabris in [19]. The main advantage in the use of a truncated correlation is the speeding up of the calculation. Indeed this kind of correlation is equal to zero where other relation might assume very small values (often negligible) and a lot of computations become trivial. The functions above, along with the assumption of conditional independence (3), allow to rewrite the posterior probability of map validity as a combination of a priori probability of map validity, the sensor characteristics, and correlation among maps validity.

A detailed calculation of this function can be found in [19], while eq. (7) shows the final formula:

$$P\left[m(p_i)|\bar{\mathcal{S}}\right] = \frac{\sum_{\{V,I\}^N}\Big[P\left[m(p_i)\mid m(p_{j1})\dots m(p_{jn})\right]\cdot}{\sum_{\{V,I\}^N}\prod_{i=1}^{N}\Big[P\left[s(p_i)\mid m(p_i)\right]\cdot}$$
$$\frac{\cdot\prod_{h=j1}^{hn}P\left[s(p_h)\mid m(p_h)\right]\cdot P\left[m(p_{j1})\dots m(p_{jn})\right]\Big]}{\cdot P\left[m(p_{j1})\dots m(p_{jn})\right]\Big]} \tag{7}$$

where $\bar{\mathcal{S}}$ is a set of labels over all the regions that may also (but not necessarily have to) contain the label given to the i^{th} region and where the subscript $\{V,I\}^N$ represent all the possible combination of map validity values that the vector $m(p_{j1}),\dots,m(p_{jn})$ can assume. The inclusion of the sensor characteristics in the approach allows to take into account the differences in precision and accuracy of the different sensors. This can be done simply by varying the value of P$[s(p_i)\mid m(p_i)]$ according to the position considered and its specific accuracy.

A. Simulation 1 - sparse measurements over a grid

In order to evaluate the effect of the correlation function, we consider a case where measurements are very sparse. The environment is composed of a grid of 2500 regions. Each region is a square having length of $1\,\mathrm{m}$. Sensors only provide labels for 7 regions. Overall only $0.3\,\%$ of the regions are observed. We consider a prior distribution where for each region the probability of being valid is equal to 0.5. Figure 2 shows the regions in the map along with the labels associated by the labelling system. Although we understand that this case is highly unrealistic, it allows to better assess the effect of the correlation function on the computation of the posterior distribution.

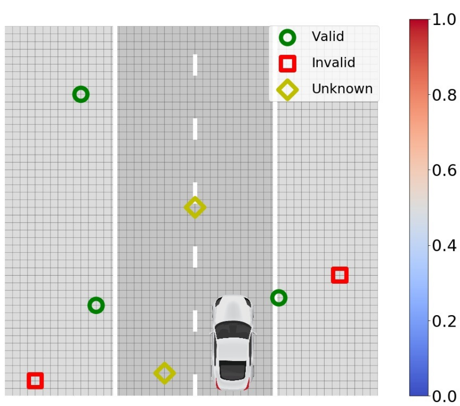


Fig. 2: Prior distribution of valid map. Each region is a square of length $1\,\mathrm{m}$. Labels from the sensors are also shown.

Figure 3 shows the posterior distribution of the map being valid for different values of β_{corr}. Since the exemplary correlation function eq. (4) only depends on the distance between regions, the probability of the map being valid or invalid changes almost equally along every direction from each measurement. Furthermore, since the correlation function is zero after a predefined distance, some regions of the map are not affected by the measurements and keep their prior value, e.g., regions below the legend in fig. 3 (a) and (b). Finally, since assigning the Unknown label to any region is independent of its prior value, regions, labelled as Unknown, preserve their prior value.

B. Simulation 2 - effect of a non-constant prior distribution

In this case we consider a prior distribution which changes over space: regions on the road have high probability of being valid, whereas regions outside the road have low probability of being valid. This example reflects the case where a backend system can communicate to vehicles the likelihood that certain regions of the map are valid, e.g., based on data collected from a fleet. We expect in this case that the fleet often assess the validity of regions in the road, but other regions might only be sporadically be observed, and hence receive low probability of being valid due to conservative measures.

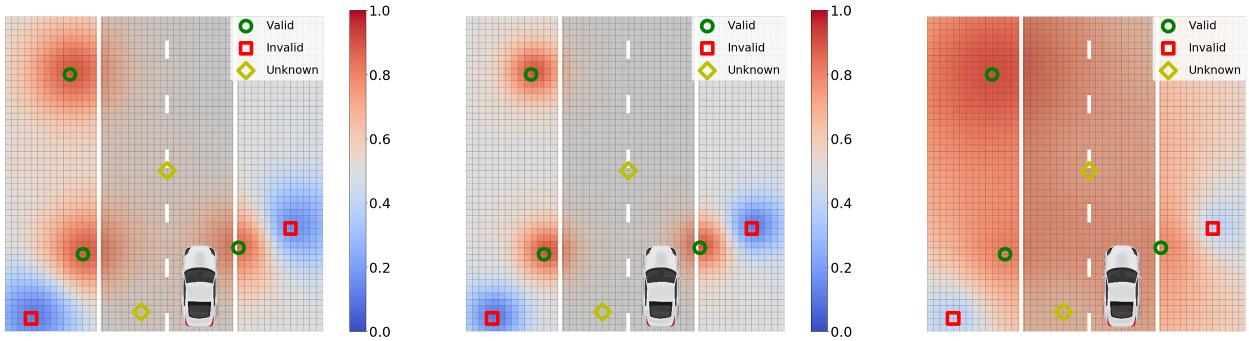
Figure 4a show the prior distribution used in this example and fig. 4b shows the obtained posterior. A value of β_{corr} of 0.3 was used.

VI. Considering low-level data

The proposed approach can be extended to account for sensors that are not equipped with classification systems able to assign validity labels to regions and only provide low-level data, such as the observed position of lanes and traffic signs. We consider the case where an ADV is equipped with three sensors: camera, LIDAR, and RADAR. The camera is located behind the windscreen and can provide information about the position of road markings and traffic signs within a horizontal FOV of $120°$. The LIDAR is placed in the front bumper of the vehicle and provides information about the position of road markings and traffic signs within a horizontal FOV of $120°$. The RADAR is placed close to the LIDAR and provides information only about the position of traffic signs within a horizontal FOV of $100°$. Camera and LIDAR observe both traffic signs and lane marking, whereas RADAR observes the position of traffic signs only.

The equation modeling the characteristic of the sensors needs to be modified in order to describe the probability of observing the traffic sign within a certain volume and this probability is different for the camera, LIDAR, and RADAR. Also, camera and LIDAR define the probability of observing lanes within certain areas of the road. The shape of eq. (7) remains instead unchanged. A thorough discussion of this example is given by Fabris in [19].

The map is divided into four regions based on the overlap of the FOV of the sensor, as shown in fig. 5. The distance between regions is defined as the distance of their centroids. All sensors measure the traffic sign in a position close to where it is expected and hence all measurements contribute to a high probability of map valid in region 1. Furthermore, both camera and LIDAR observe the lanes positioned close to what is expected from the map. In region 2 the camera observes the lanes in positions relatively far from what is

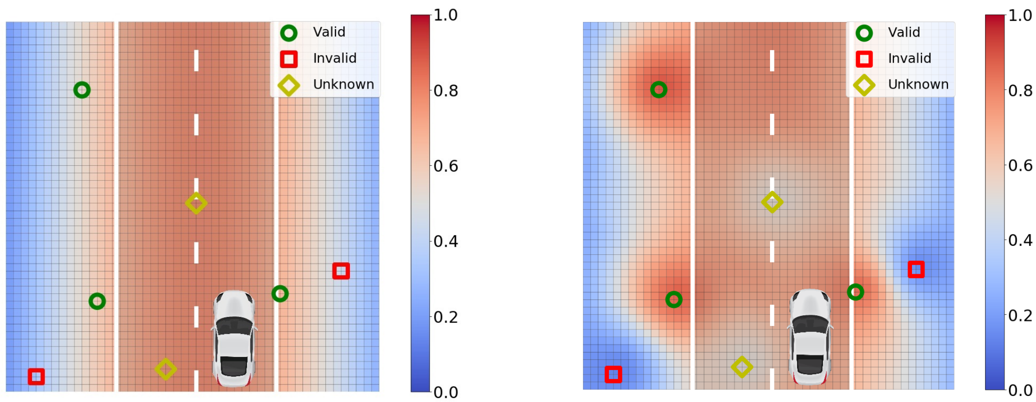


(a) $\beta_{corr} = 0.3$

(b) $\beta_{corr} = 0.5$

(c) $\beta_{corr} = 0.13$

Fig. 3: Posterior distribution of the map validity with different values of the correlation parameter β_{corr}.



(a) Prior distribution.

(b) Posterior distribution.

Fig. 4: Distribution of the map validity when prior distribution is not constant over space.

described in the map. In region 3 the camera observes the lanes in positions that are relatively close to what is described in the map. Finally, no measurements are available for region 4. The a priori distribution of map validity is constant and set to 0.5.

Figure 6 shows the effect of the parameter β_{corr} on the validity probability of the four regions. As β_{corr} tends to 0, the probability of all regions converge to the same value since extremely high correlation among regions is considered. As β_{corr} increases, the effect of the correlation decreases, and the validity of the regions only depend on the local measurements and on the prior distribution. In this case, region 1 and 3 tends to a high probability of being valid, since measurements fit well with the expected value. The probability of region 2 being valid instead tends to 0, since the lane observed in this region by the camera does not match well against what is expected from the map. As β_{corr} increases, the validity of area 4 tends to the prior distribution since no measurements are available for this region.

VII. Discussion and Future Research Proposals

A. Resume and Results

This paper shows a model-based approach to OMV, which is based on the following main ideas: (i) a probabilistic representation of the information coming from the sensors to determine the validity of the map and, (ii) the spatial-temporal interpolation of the knowledge in order to evaluate the probability of validity even in areas where no evidence from sensors is available.

The representation of experimental data in the form of probabilities allows us to completely decouple the proposed approach from the physical characteristics of the sensors. In this way, the information can come not only from specific sensors placed on the vehicle but also from the result of sensor fusion algorithms. A few examples have been discussed in this paper. As discussed in Fabris [19], other cases can also be considered, e.g., compute the validity of the map based on the observed behaviors of other vehicles.

The proposed approach allows considering the temporal and spatial distance of measurements. When observations are sparse, the result of the posterior distribution strongly depends on the selection of the correlation function. The correlation function may not be isotropic but it can develop itself in one or more preferential directions. For example, in the case of a construction site sign, the correlation function may be chosen to only affect regions behind the sign.

At this stage, it is unclear which functions are most suitable for representing realistic correlation among regions

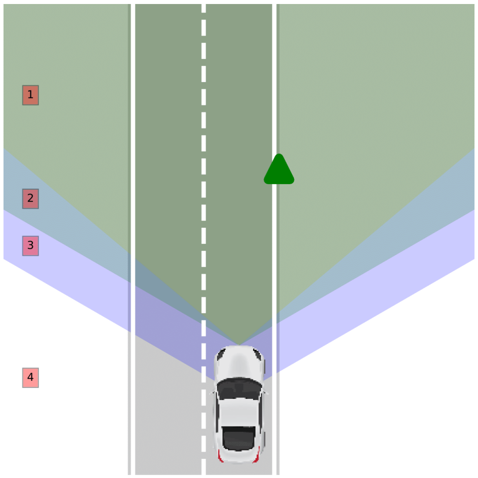


Fig. 5: Areas observed by the sensors and their FOV. The green triangle represents a traffic sign.

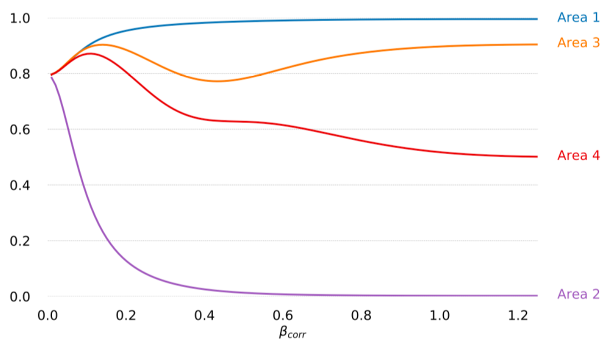


Fig. 6: Effect of β_{corr} on the posterior distributions of the four areas.

of map validity and how to tune the different parameters used in the algorithm.

B. Future Work

Possible fields of future research could be the use of more detailed sensor models in order to obtain more information from the surrounding environment and with better accuracy. More precisely, this research can be approached to an in-depth study of the conversion of measurable information directly from the sensors, in probability that can be used to validate the high resolution map. Another possible field of development can certainly concern the segmentation of the environment in order to limit the computational complexity while guaranteeing good performances. By developing an algorithm capable of subdividing the environment optimally (for example, by segment the road used by the vehicle in a more refined way and the surrounding environment in a more coarse manner) could undoubtedly have improvements both in terms of performance and computational complexity. In order to improve the robustness of the algorithm, approaches for including effects of localization error and uncertainty should also be researched.

Our research work is still at its infancy and a thorough comparison against other algorithms is still missing. Moreover, it might be interesting to define approaches that combine the proposed correlation-based approach with data-driven approaches, with the goal of simplifying the tuning process of the parameters used in the models.

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